Why do you Invert and Multiply?

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Resources


A novel by James A. Michener, released in 1985, describes TEXAS as “a land of sprawling diversity and unparalleled richness, a dazzling chapter in the history of our nation, a place like no other on earth.” Michener provided an “epic saga spanning four centuries…from the age of conquistadors to the present day.”

The little town of Larkin, population reduced to a sane 3,673, now boasted seven millionaires…

Some of the new millionaires spent their money conspicuously, and Dewey often spoke of one who had never fully appreciated the intricacies of the oil game: “When I went to talk him into leasing us his land, I offered him the standard one-eighth royalty, but he said, ‘I know you city slickers. I want one-tenth,’ so, after considerable pressure I surrendered.
Sometime later he came to me, all infuriated: ‘You dirty scoundrel, you cheated me.’ I said, ‘Hold on a minute. You set the royalty, not me.’ and he said, ‘I know that ….

…..But Gulf offered me **one-twelfth**!’
Me No Make Numbers Good

Because we haven’t had to do math in aeons, the mental strain it takes to dig through the piles of mildew in one’s brain to retrieve given equations is brutal. *How do you divide fractions again? Don’t you flip the top number and the bottom number or something? And what’s the top number called? The ruminator? The kilometer?*

Problem: When I got home last night, I found my puppy not feeling very well. So, I took her to the veterinarian.

Our vet said to give our dog some medicine. She gave us 15 tablets.

Because our dog is very large (100 pounds) the vet said to give the dog $\frac{12}{3}$ tablet each day. For how many days will the medicine last?
15 tablets, $1\frac{2}{3}$ per day

Fig. 3.2. Stephanie’s work

You have $6\frac{2}{3}$ candy bars. You had 16 people you wanted to give each $\frac{2}{3}$ of a candy bar. How many people didn't get the candy bar?

\[
6\frac{2}{3} \div \frac{2}{3} = 10
\]

\[
10 \times \frac{2}{3} = 6\frac{2}{3}
\]

8 didn't get any.

Fig. 8.2. Student-posed word problem for division with fractions

Division

Measurement (Repeated subtraction):
“There are six cookies. If I give each child two cookies, how many children will get cookies?” The question: How many groups of 2?

Partition (Sharing):
“There are six cookies. If I share them equally between two children, how many cookies will each child get?” The question: How many in each of 2 groups?

Determination of a Unit Rate

As the Inverse of Multiplication

As the Inverse of a Cartesian Product

Etc.
You are not to wonder why...

Just invert and multiply!
What we would “expect” them to do...

\[
\frac{3}{4} ÷ \frac{3}{8} = \frac{4}{8} = \frac{3 \times 8}{4 \times 3} = \frac{24}{12} = 2
\]

\[
\frac{3}{4} ÷ \frac{3}{8} = \frac{3 \times 8}{4 \times 3} = \frac{24}{12} = 2
\]
“Well, I know I have to inverse them first, then multiply…”

\[
\frac{3}{4} ÷ \frac{3}{8} = \frac{3\times3}{8\times4} =
\]
“You have to invert and multiply...”

\[
\frac{3}{4} \div \frac{3}{8} = \frac{4}{3} \times \frac{8}{3} =
\]
I know
I have to invert and multiply,
but I just never remember which one gets flipped...
Premature symbolism...

...leads to symbolic knowledge that students cannot connect to the real world, resulting in virtual elimination of any possibility for students to develop number and operation sense.

Symbolic knowledge...

...that is not based on understanding is “highly dependent on memory and subject to deterioration”.

\[ \frac{3}{4} \]

or

\[ \frac{3}{8} \]

\[ \frac{3\sqrt{3}}{8} \]

or

\[ \frac{3 \div 3}{4} \]

\[ \frac{5}{x+2} \]

\[ \left( \frac{x+2}{5} \right)^2 \]
Why don’t we just divide when asked to divide?

\[
\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \div 1 = \frac{3}{1} = 3
\]
TRY IT!

\[
\begin{align*}
\frac{5}{6} \div \frac{1}{6} &= \, ? \\
\frac{5}{6} \div \frac{1}{3} &= \, ? \\
\frac{3}{8} \div \frac{1}{4} &= \, ? \\
\frac{3}{4} \div \frac{3}{8} &= \frac{3}{4} \times \frac{2}{3} \div 3 = \frac{6}{8} \div \frac{3}{8} = \, ? \\
\frac{4}{5} \div \frac{2}{3} &= \frac{4}{5} \times \frac{3}{2} \div \frac{5}{3} = \frac{12}{15} \div \frac{10}{15} = \, ?
\end{align*}
\]
Common Denominator

\[
\frac{1}{3} \div \frac{1}{4} = \frac{\left[ \frac{4}{3} \right]}{\left[ \frac{4}{3} \right]} = \frac{4}{12} \div \frac{3}{12} = \frac{4}{4} \div \frac{3}{3} = \frac{1}{1} = \frac{4}{3}
\]

\[
\frac{4}{12} \div \frac{3}{12} = \frac{4}{3} \div \frac{3}{3} = \frac{4}{3}
\]
“Convenient” Denominator

\[
\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \left( \frac{4}{4} \right) \div \frac{1}{4} = \frac{4}{12} \div \frac{1}{4} =
\]

\[
\frac{4}{12} \div 4 = \frac{4}{3}
\]

\[
\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \left( \frac{3}{3} \right) \div \frac{1}{3} = \frac{12}{15} \div \frac{1}{3} =
\]

\[
\frac{12}{15} \div 3 = \frac{12}{5}
\]

\[
\frac{5}{6} \div \frac{1}{8} = \frac{5}{6} \left( \frac{4}{4} \right) \div \frac{1}{8} = \frac{20}{24} \div \frac{1}{8} =
\]

\[
\frac{20}{24} \div 8 = \frac{20}{3}
\]
\[
\frac{2}{2} = 1 \\
\frac{3}{3} = 1
\]
Rational Expression

\[
\frac{(x+2)^2}{5} \div \frac{5}{x+2} = \frac{(x+2)^2}{5} \div \frac{5}{5} = \frac{x+2}{1}
\]
So, why **DO** we teach kids to “Invert and Multiply”?

\[
\frac{3 \div 3}{4 \div 8} = \frac{3 \times 8}{4 \div 3} = 2
\]

\[
\frac{3 \div 3}{4 \div 8} = \frac{3 \div 3}{4 \div 8} = \frac{1}{2}
\]

\[
\frac{3\left(\frac{2}{2}\right) \div 3}{4 \div \frac{8}{8}} = \frac{6 \div 3}{8 \div 8} = \frac{2}{1}
\]
So, why *DO* we teach kids to “Invert and Multiply”?

\[
\frac{5}{11} \div \frac{3}{7} = \frac{5}{11} \times \frac{7}{3} = \frac{35}{33}
\]
OK

– So, tell me....

When in your real life have you ever needed to divide five-elevenths by three-sevenths?
Remember 9-year old Stephanie and her big dog?

\[
\frac{\frac{45}{3}}{\frac{5}{3}} = 9
\]

Many children, even those who know the “invert and multiply” algorithm, prefer to use only common denominators when dividing a fraction by a fraction (Sharp, 1998 NCTM Yearbook, p. 203). They find the concept of common denominators familiar from earlier work with adding or subtracting.
1. \( \frac{3}{2} \div \frac{1}{2} = ? \)

8. \( 1\frac{1}{3} \div \frac{2}{3} = ? \)

10. Each bow requires one-half yard of ribbon. If Sue has two and one-half yards of ribbon, how many bows can she make?
10. Each bow requires one-half yard of ribbon. If Sue has two and one-fourth yards of ribbon, how many bows can she make?
1. \[ \frac{1}{3} \div \frac{1}{2} = ? \]

8. \[ 1\frac{1}{2} \div \frac{2}{3} = ? \]

10. Each bow requires one-half yard of ribbon. If Sue has two and one-third yards of ribbon, can she make five bows?